

数学参考答案

一、选择题(本题有 10 小题,每小题 4 分,共 40 分)

题号	1	2	3	4	5	6	7	8	9	10
答案	C	B	B	A	D	A	B	C	D	C

二、填空题(本题有 6 小题,每小题 5 分,共 30 分)

$$11. a(a-3) \quad 12. 37 \quad 13. \begin{cases} x=3, \\ y=1. \end{cases} \quad 14. 46 \quad 15. (32\sqrt{2}+16) \quad 16. \frac{3\sqrt{7}}{2}$$

三、解答题(本题有 8 小题,共 80 分)

17.(本题 10 分)

$$\text{解}(1) \sqrt{20} + (-3)^2 - (\sqrt{2}-1)^0$$

$$= 2\sqrt{5} + 9 - 1 = 2\sqrt{5} + 8.$$

$$(2)(2+m)(2-m) + m(m-1)$$

$$= 4 - m^2 + m^2 - m = 4 - m.$$

18.(本题 8 分)

$$\text{解}(1) \text{由题意,得 } \frac{72}{360} \times 100\% = 20\%.$$

答:“非常了解”的人数的百分比是 20%.

$$(2) \text{由题意,得 } 1200 \times \frac{72+108}{360} = 600(\text{人}).$$

答:估计对“垃圾分类”知识达到“非常了解”和“比较了解”程度的学生共有 600 人.

19.(本题 8 分)

(1)证明: $\because AD \parallel BC$, 即 $AD \parallel BF$,

$$\therefore \angle 1 = \angle F, \angle D = \angle 2,$$

$$\because DE = CE, \therefore \triangle ADE \cong \triangle FCE.$$

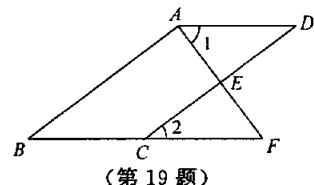
(2)解: $\triangle ADE \cong \triangle FCE$,

$$\therefore AE = EF = 3.$$

$$\because AB \parallel CD, \therefore \angle AED = \angle BAF = 90^\circ,$$

在 $\square ABCD$ 中, $AD = BC = 5$,

$$\therefore DE = \sqrt{AD^2 - AE^2} = 4, \therefore CD = 2DE = 8.$$

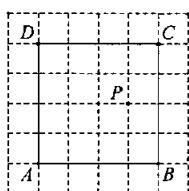


(第 19 题)

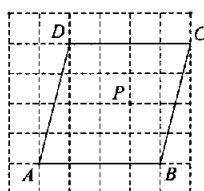
20.(本题 8 分)

解(1)画法不唯一,如图①,②,③等.

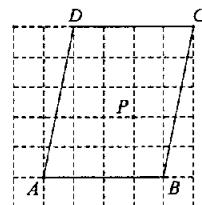
(2)画法不唯一,如图④,⑤,⑥等.



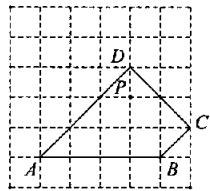
①



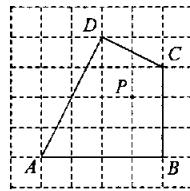
②



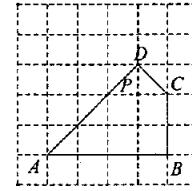
③



④



⑤



⑥

21.(本题 10 分)

(1)证明连结 DE .

$\because BD$ 是 $\odot O$ 的直径,

$$\therefore \angle DEB = 90^\circ.$$

$\because E$ 是 AB 的中点,

$$\therefore DA = DB,$$

$$\therefore \angle 1 = \angle B,$$

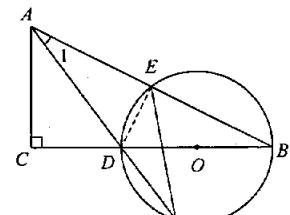
$$\because \angle B = \angle F,$$

$$\therefore \angle 1 = \angle F.$$

(2)解: $\angle 1 = \angle F$,

$$\therefore AE = EF = 2\sqrt{5},$$

$$\therefore AB = 2AE = 4\sqrt{5}.$$



(第 21 题)

在 $\text{Rt}\triangle ABC$ 中, $AC = AB \cdot \sin B = 4$,

$$\therefore BC = \sqrt{AB^2 - AC^2} = 8.$$

设 $CD = x$, 则 $AD = BD = 8 - x$.

由勾股定理, 得 $AC^2 + CD^2 = AD^2$,

$$\text{即 } 4^2 + x^2 = (8 - x)^2,$$

解得 $x = 3$.

$$\therefore CD = 3.$$

22. (本题 10 分)

$$\text{解(1)} \frac{15 \times 40 + 25 \times 40 + 30 \times 20}{100} = 22 \text{ (元/千克).}$$

答: 该什锦糖每千克 22 元.

(2) 设加入丙种糖果 x 千克, 则加入甲种糖果 $(100 - x)$ 千克, 由题意, 得

$$\frac{30x + 15(100 - x) + 22 \times 100}{200} \leq 20, \quad \text{解得 } x \leq 20.$$

答: 最多可加入丙种糖果 20 千克.

23. (本题 12 分)

解(1) ∵ 抛物线的对称轴是 $x = \frac{m}{2}$,

$$\therefore AC = m,$$

$$\therefore BE = 2AC = 2m.$$

(2) 当 $m = \sqrt{3}$ 时, 点 D 落在抛物线上. 理由如下:

$$\because m = \sqrt{3},$$

$$\therefore AC = \sqrt{3}, BE = 2\sqrt{3}.$$

把 $x = 2\sqrt{3}$ 代入 $y = x^2 - \sqrt{3}x - 3$, 得

$$y = (2\sqrt{3})^2 - \sqrt{3} \times 2\sqrt{3} - 3 = 3.$$

$$\therefore OE = 3 = OC.$$

∵ $\angle DEO = \angle ACO = 90^\circ$, $\angle DOE = \angle AOC$,

$\therefore \triangle OED \cong \triangle OCA$,

$$\therefore DE = AC = \sqrt{3}, \quad \therefore D(-\sqrt{3}, 3).$$

把 $x = -\sqrt{3}$ 代入 $y = x^2 - \sqrt{3}x - 3$, 得

$$y = (-\sqrt{3})^2 - \sqrt{3} \times (-\sqrt{3}) - 3 = 3.$$

∴ 点 D 落在抛物线上.

(3) ① 如图 2, 当 $x = 2m$ 时, $y = 2m^2 - 3$, $OE = 2m^2 - 3$.

∵ $AG \parallel y$ 轴,

$$\therefore EG = AC = \frac{1}{2}BE,$$

$$\therefore FG = \frac{1}{2}OE.$$

$$\therefore S_{\triangle DOE} = S_{\triangle BGF}, \text{ 即 } \frac{1}{2}DE \cdot OE = \frac{1}{2}BG \cdot FG,$$

$$\therefore DE = \frac{1}{2}BG = \frac{1}{2}AC.$$

∵ $\angle DOE = \angle AOC$, $\therefore \tan \angle DOE = \tan \angle AOC$,

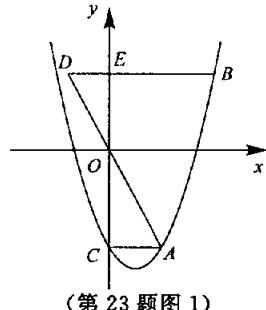
∴ $\angle DEO = \angle ACO = \text{Rt} \angle$,

$$\therefore \frac{DE}{OE} = \frac{AC}{OC},$$

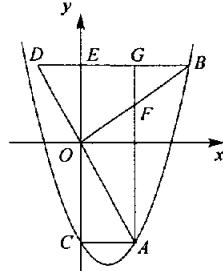
$$\therefore OE = \frac{1}{2}OC,$$

$$\therefore 2m^2 - 3 = \frac{3}{2}, \therefore m = \frac{3}{2}.$$

② m 的值是 $\frac{3\sqrt{2}}{2}$.



(第 23 题图 1)



(第 23 题图 2)

24. (本题 14 分)

(1) 证明如图 1, 设 $\odot O$ 切 AB 于点 P , 连结 OP , 则 $\angle OPB = 90^\circ$.

\because 四边形 $ABCD$ 是菱形,

$$\therefore \angle ABD = \frac{1}{2} \angle ABC = 30^\circ,$$

$$\therefore BO = 2OP = 2OM.$$

(2) 解如图 2, 设 GH 交 BD 于点 N , 连结 AC , 交 BD 于点 Q .

\because 四边形 $ABCD$ 是菱形,

$$\therefore AC \perp BD.$$

$$\therefore BD = 2BQ = 2AB \cdot \cos \angle ABQ = \sqrt{3} AB = 18.$$

设 $\odot O$ 的半径为 r , 则 $OB = 2r, BM = 3r$.

$\because EF > HE$, \therefore 点 E, F, G, H 均在菱形的边上.

(I) 如图 2, 当点 E 在边 AB 上时.

$$\text{在 } \triangle BEM \text{ 中}, EM = BM \cdot \tan \angle EBM = \sqrt{3} r.$$

$$\text{由对称性, 得 } EF = 2EM = 2\sqrt{3} r,$$

$$DN = BM = 3r,$$

$$\therefore MN = 18 - 6r,$$

$$\therefore S_{\triangle EFGH} = EF \cdot MN = 2\sqrt{3} r(18 - 6r) = 24\sqrt{3},$$

$$\text{解得 } r_1 = 1, r_2 = 2.$$

当 $r = 1$ 时, $EF < HE$,

$\therefore r = 1$ 不合题意, 舍去.

当 $r = 2$ 时, $EF > HE$,

$$\therefore r = 2. \text{ 此时 } BM = 3r = 6.$$

(II) 如图 3, 当点 E 在边 AD 上时.

$$\text{由对称性, 得 } BM = 3r = 18 - 6 = 12,$$

$$\therefore r = 4.$$

综上所述, $\odot O$ 的半径是 2 或 4.

(3) 解设 GH 交 BD 于点 N , $\odot O$ 的半径为 r , 则 $BO = 2r$.

当点 E 在边 BA 上时, 显然不存在 HE 或 HG 与 $\odot O$ 相切.

(I) 当点 E 在边 AD 上时.

(i) 如图 4, 当 HE 与 $\odot O$ 相切时.

$$\text{则 } EM = r, DM = \sqrt{3} r,$$

$$\therefore 3r + \sqrt{3} r = 18,$$

$$\therefore r = 9 - 3\sqrt{3},$$

$$\therefore BO = 2r = 18 - 6\sqrt{3}.$$

(ii) 如图 5, 当 HG 与 $\odot O$ 相切时.

由对称性, 得

$$ON = OM, BN = DM,$$

$$\therefore BO = \frac{1}{2} BD = 9.$$

(II) 当点 E 在边 AD 的延长线上时.

(i) 如图 6, 当 HG 与 $\odot O$ 相切时, $MN = 2r$.

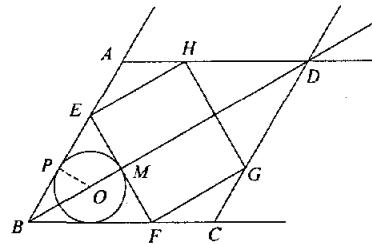
$$\therefore BN + MN = BM = 3r,$$

$$\therefore BN = r,$$

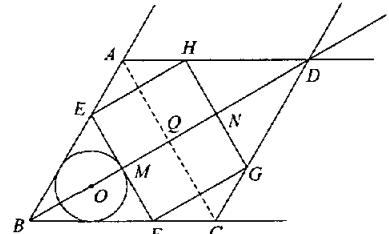
$$\therefore DM = \sqrt{3} FM = \sqrt{3} GN = BN = r,$$

$\therefore D$ 与 O 重合.

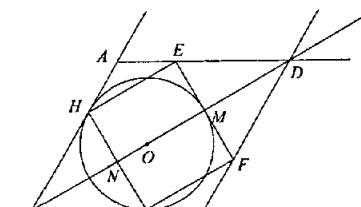
$$\therefore BO = BD = 18.$$



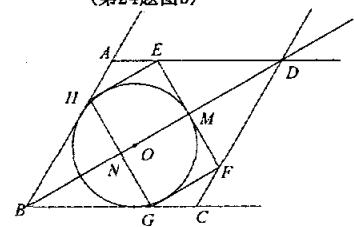
(第 24 题图 1)



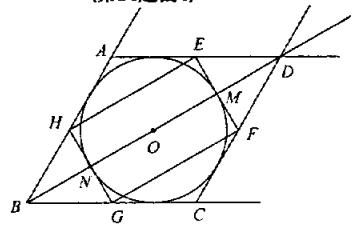
(第 24 题图 2)



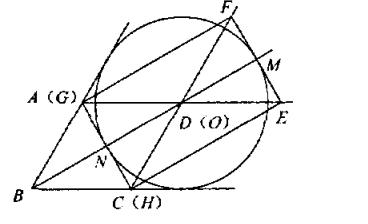
(第 24 题图 3)



(第 24 题图 4)



(第 24 题图 5)



(第 24 题图 6)

(ii) 如图 7, 当 HE 与 $\odot O$ 相切时.

$$\text{则 } EM=r, DM=\sqrt{3}r,$$

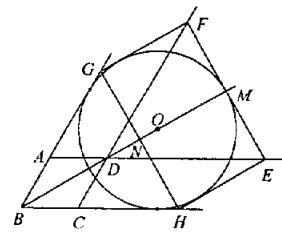
$$\therefore 3r-\sqrt{3}r=18,$$

$$\therefore r=9+3\sqrt{3},$$

$$\therefore BO=2r=18+6\sqrt{3}.$$

综上所述, 当 HE 或 HG 与 $\odot O$ 相切时,

BO 的长为 $18-6\sqrt{3}$ 或 9 或 18 或 $18+6\sqrt{3}$.



(第24题图7)